

### 3.3.2

Find an example of a bounded discontinuous function  $f: [0, 1] \rightarrow \mathbb{R}$  that has neither an absolute minimum nor an absolute maximum

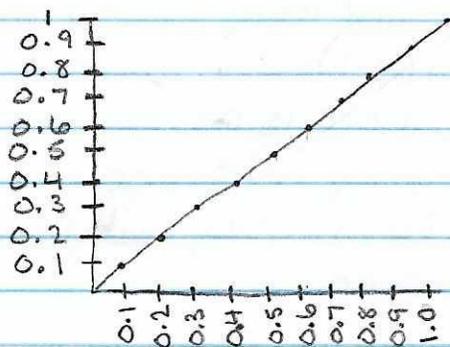
**Recall:**

Bounded:  $0 \leq f(x) \leq 1$  for all  $x$

Discontinuous: There is a hole or "jump" in the graph

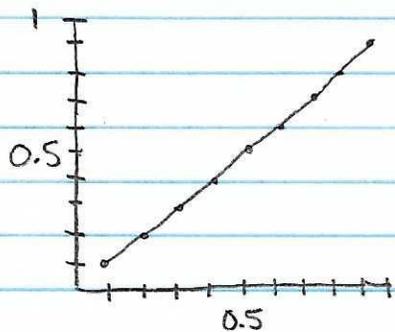
**Try:**

① Make a graph between  $[0, 1]$

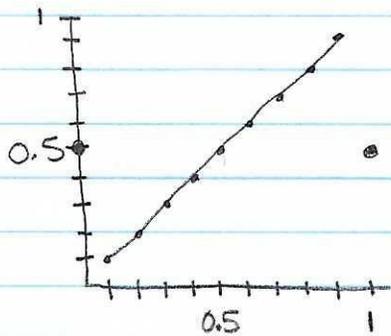


$$f(x) = x \text{ if } x \in [0, 1]$$

② This won't work because it has a max and min. How do we make it not have a max and min? What if as  $x$  increases, the function never reaches 1? What if as  $x$  decreases, the function never reaches 0? Re-graph it without 0 or 1.



- ③ However, we can't disclude 0 and 1 because the question states it is on a closed interval. This is easy to fix. Since the graph is supposed to be discontinuous, we can place these points anywhere.  
(except at 0 and 1)



- ④ Now, as  $f(x)$  approaches 1, the max approaches 1. It can go on for infinity. Ex:  $f(0.9) = 0.9$ ,  $f(0.99) = 0.99$ ,  $f(0.999) = 0.999$ . So, there is no real max since  $f(x)$  is always increasing, and once it hits  $f(1)$ , the answer is 0.5. Definitely not the max. The same is true for the minimum. So,

$$f(x) = \begin{cases} 0.5 & \text{if } x=0 \\ x & \text{if } x \in (0,1) \\ 0.5 & \text{if } x=1 \end{cases}$$